



A Bayesian multivariate Poisson regression model of cross-category store brand purchasing behavior

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Abstract

The availability of cross-category transaction data in the retailing industry has enabled the investigation of interdependence in consumer purchase behavior across product categories. In this paper, we develop a multivariate count model to uncover and predict the pattern of cross-category store brand purchasing behavior. The proposed multivariate Poisson regression model, which we estimate using a Bayesian approach, provides flexibility in capturing cross-category correlations for sparse multivariate purchase data associated with infrequently purchased categories or purchasing in retail outlets such as warehouse clubs. We compare the goodness-of-fit of the proposed Poisson regression model with alternate benchmark models using customer purchase records across five product categories from a national warehouse club and find that the proposed model provides a superior fit. We also carry out a profitability analysis to illustrate the use of the model in planning cross-promotions.

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1. Introduction

A sound understanding of cross-category consumer purchase behavior should be of interest to retailers who seek to maximize their profit by jointly promoting selected products across categories. The analysis of cross-category consumer traits, such as price sensitivity and store brand proneness, generally involves the development of multi-category choice models (e.g., Ainslie and Rossi, 1998; Hansen et al., 2006). However, the estimation of multivariate choice models to capture cross-category correlations becomes difficult when the data contain sparse purchases per household, such as in infrequently purchased product categories or for shoppers in retail outlets such as wholesale stores and warehouse clubs. The latter groups of consumers tend to buy in bulk, albeit infrequently, to take advantage of the price discounts offered by wholesale stores and warehouse clubs. The average shopping frequency per household in warehouse channels is less

than one-sixth the frequency in grocery channels (Retail Merchandiser, 2004).

In this paper, we develop a multivariate Poisson regression model to uncover and predict the pattern of cross-category store brand purchasing behavior using member transaction data across five product categories from a national warehouse chain. The proposed model captures cross-category correlations with minimum data requirements and is well-suited to the analysis of multivariate count data (non-negative integers). Furthermore, the multivariate Poisson model supports both negative and positive correlations and allows the specification and testing of hypotheses pertaining to independence and complete dependence in consumer purchasing behavior for different categories (Aitchison and Ho, 1989). Finally, the proposed approach allows us to partial out the effects of the price spread between the store brand and national brands as well as the impact of selected customer characteristics, and thus, effectively captures cross-category interrelations with regard to store brand purchasing.

We use the Monte Carlo Markov chain (MCMC) technique within the Bayesian framework to estimate the

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proposed multivariate Poisson regression model. This Bayesian approach estimates the joint distribution of multiple counts on the basis of customers' purchases of a given store brand in multiple categories. The application of the Bayesian multivariate mixed Poisson model helps address the following research questions of managerial interest. To what extent is a store-brand buyer in one category prone to purchase the same store brand in a *related* category (e.g., multivitamins and ibuprofen)? Further, how likely is a store-brand buyer in one category, say, laundry detergent to purchase the same store brand in an *unrelated* category, such as diapers? Empirical results from an analysis of consumer purchases of a store brand across five different product categories reveal a strong interrelationship in store brand purchasing behavior across related as well as unrelated product categories. Furthermore, we find that the price spread between the store brand and national brands and store visit frequency relate positively to consumers' propensity to purchase a store brand.

To the best of our knowledge, this is the first application of multivariate count regression models in a marketing setting. As mentioned, a feature of the proposed multivariate Poisson regression model is that it can be estimated with sparse multivariate purchase data typically available to warehouse clubs while accounting for the effects of marketing variables. Further, the Bayesian approach used to estimate the proposed model has the flexibility to handle high-dimension data and can accommodate a general correlation structure. It is also worthwhile to mention that unlike previous analyses of purchase interdependence based on count data, the proposed model estimates cross-category correlation in consumer propensity to purchase the same brand across different categories. It thus offers a new way to extract information from cross-category transaction data and provides useful insights for managers whose brands have offerings in multiple categories.

Substantively, we hope that our empirical study will enhance our understanding of consumer purchase behavior in wholesale stores and warehouse clubs. Most cross-category analyses that use panel data focus solely on the supermarket retail format. As warehouse clubs continue to adopt aggressive branding strategies and extend their franchises of premium-quality store brands to diverse categories, it becomes ever more important to understand store brand purchasing behavior in this particular channel. Finally, with respect to managerial implications, our results may be useful to retail managers in planning joint promotions of store brands across categories by leveraging information on cross-category correlations. We show, later in the paper, how the proposed model may be used to assess and compare the overall profit impact of joint promotions for different category combinations.

The following section provides a brief review of alternate methods for analyzing multivariate count data and the relevant literature on store brand purchasing behavior. Then, Section 3 describes the proposed multivariate mixed

Poisson-lognormal model and the Bayesian estimation procedure. A description of the data appears in Section 4. After a discussion of the empirical results and a comparison of the proposed model with alternate benchmark models, the article concludes with a discussion of the managerial implications of the empirical results, directions for further research, and limitations of our research study.

2. Literature review and conceptual background

2.1. Alternate methods for analysis of multivariate count data

Managers with limited background knowledge of the purchasing behavior across the focal product categories can use undirected data mining approaches to explore and discover previously unknown associations or interdependent purchase patterns between product categories. To determine which products are likely to be purchased jointly, researchers use association rules, which extract information from transaction data on the basis of the frequencies of simultaneous purchases of different combinations of products. Various techniques can be used to construct measures of product interdependences such as association coefficients or frequent item sets (Brijs et al., 2004). Alternatively, researchers can employ the neural network approach to identify the interrelationships between products. This approach uses market baskets constructed as binary vectors that indicate the presence or absence of purchases in selected corresponding categories. Purchase interdependences emerge from the frequencies of output patterns and the weight vectors associated with each category (e.g. Decker and Monien, 2003). However, none of these methods considers explanatory marketing or consumer factors that may underlie the interdependence pattern, which makes it difficult to interpret and explain their results and limits their benefits for marketing decision making.

When hypotheses of plausible interrelationships between count variables can be developed from theory and prior knowledge, a more directed approach, such as multivariate regression count models, is appropriate. The correlation structure in multivariate models can be used to measure purchase interdependence. Moreover, multivariate regression models can account for explanatory variables, which help capture the true extent of the interrelationship and improve the interpretability of results. However, applications of multivariate count models are relatively rare because of the difficulties associated with their estimation procedures. Most applications to date appear in the economics literature and use Poisson distribution in the model specification.

There are several techniques that introduce correlations into multivariate Poisson models. One of the best known techniques employs a stochastic component that shares the specification of the count variables through either multivariate reduction techniques (Karlis, 2003) or the method

of mixtures (Jung and Winkelmann, 1993; Munkin and Trivedi, 1999). However, the use of an identical component restricts the effects of the factors to be the same across categories. Furthermore, this approach permits only positive correlations between two variables.

A more general method allows for separate stochastic components that are correlated for each count variable. The mixing distribution in this context can be either a discrete distribution with several support points (finite mixture) or a parametric continuous distribution (continuous mixture). The finite mixture approach can flexibly accommodate extreme and/or strongly asymmetric departures from the Poisson model (Alfo and Trovato, 2004). However, finite mixture models may inadequately represent the full extent of heterogeneity and it is difficult to estimate them with more than a half dozen or so mass points. Allenby and Rossi (1999) discuss and provide empirical evidence of the shortcomings of the finite mixture. An alternative parametric specification for the mixing distribution is the use of multivariate normal distribution, which leads to a multivariate Poisson lognormal model that can be estimated by either simulated maximum likelihood (Munkin and Trivedi, 1999) or MCMC method (Chib and Winkelmann, 2001). However, the simulated maximum likelihood method makes it difficult to extend the model beyond the case of a few outcomes, which may limit the scope of data mining applications. In contrast, Bayesian inference using the MCMC method can efficiently accommodate high-dimensional count data while still allowing the model to maintain a general correlation structure.

2.2. Store brand purchasing behavior

Consumers may be characterized by their willingness to trade off product quality for a price discount (Blattberg and Wisniewski, 1989). Consumers who place a much higher value on quality versus price generally prefer to purchase only national label brands. Despite the recent efforts on the part of private label suppliers to upgrade the quality of store brands, quality conscious consumers may feel uncertain about the quality of a store brand because of the traditional inferior quality image of private label brands. This uncertainty likely creates perceived risk and deters such consumers from trying the store brand. Perceived risk has been documented as an important factor in store brand purchasing behavior (Dick et al., 1995; Richardson et al., 1996; Batra and Sinha, 2000; Erdem et al., 2004).

Store brands attract the price sensitive segment of consumers by offering a price discount relative to national brands. The price sensitive consumers are willing to trade off product quality to obtain a significant price discount. Empirical evidence documents the general propensity of price sensitive consumers to purchase store brands across categories (Ailawadi et al., 2001). In some categories, an intermediate segment of consumers experience a trade off

between quality and price that is less extreme (Corstjens and Lal, 2000). If the quality difference is perceived as small or within an acceptable range, these consumers may consider purchasing a lower quality brand for a price discount (Bronnenberg and Wathieu, 1996; Sethuraman and Cole 1999). For such consumers, the appeal of store brands depends primarily on their perceptions of the quality of the store brands relative to that of national brands (Hoch and Banerji, 1993).

Recently, retailers have been undertaking significant promotional efforts designed to enhance the quality perceptions of the store brands they sell exclusively to shoppers at their stores. Large retailers and warehouse clubs promote store brands through in-store demonstrations, features of the store-brand items in newspaper ads, mailed flyers, and newsletters sent to club members. In essence, retailers have begun to market store brands just as the major national brands do. Wernerfelt (1988) suggests that an umbrella brand name can be used as a bond for the quality of other products with the same name; so if consumers have a positive experience with a private label brand in one category, they may update their overall perceptions of the umbrella brand. In turn, those positive perceptions of the umbrella brand may be transferred from one category to another and thereby reduce consumers' perceived risk in purchasing the umbrella brand in a new product category (Montgomery and Wernerfelt, 1992).

In summary, store brands appeal to both price sensitive consumers and the intermediate segment of consumers who are willing to trade off quality for a price discount. Price conscious consumers thus may exhibit general cross-category purchase propensities toward store brands (Cunningham et al., 1982; Dick et al., 1995; Ailawadi et al., 2001). In addition, the use of the same brand name and consistent positioning across categories may enhance the store brand's appeal on the quality dimension, which may attract the intermediate segment of consumers. Therefore, there should be a significant positive association between the purchases of a premium quality umbrella store brand across even dissimilar categories, though less than that across categories with similar attributes.

3. Model and estimation

3.1. Model

An essential premise of the cross-category store brand purchasing model is that it assumes a consumer i has a latent propensity b_{ij} to purchase the focal store brand in a given product category j . Consumer i 's total number of purchases of the store brand during a certain period of time (e.g., a year) in the category is a function of b_{ij} , as well as other observable marketing variables, such as the price spread between the focal store brand and the national brands and individual consumer characteristics. The parameter b_{ij} may be interpreted as consumer i 's underlying tendency to purchase the store brand in category j , because

it captures the impact of consumer-specific characteristics and category-specific unobserved variables on the consumer's propensity to purchase the store brand. The association in consumers' latent propensities to purchase the store brand in different categories causes store brand purchase counts to correlate across categories.

Consider n consumers who have an opportunity to purchase the store brand in each of J product categories. Let y_{ij} denote consumer i 's total number of purchases of a store brand in category j during the sample period. Then, y_{ij} may be assumed to be distributed as a Poisson random variable with the mean purchase rate parameter μ_{ij} . That is,

$$P(y_{ij}) = \frac{\exp(-\mu_{ij})\mu_{ij}^{y_{ij}}}{y_{ij}!}, \tag{1}$$

for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, J$. For each category, consumer i has a unique, category-specific mean store-brand purchase rate μ_{ij} , which depends on the consumer's latent propensity to purchase the store brand in category j , b_{ij} , and various observable marketing variables and individual customer characteristics X_{ij} . Thus, μ_{ij} is specified as

$$\mu_{ij} = c_{ij} \exp[X'_{ij}\beta_j + b_{ij}], \tag{2}$$

where c_{ij} is consumer i 's total number of purchases in the product category, X_{ij} represents a $K \times 1$ explanatory variable vector and β_j is a $K \times 1$ vector of corresponding parameters, including an intercept term β_0 . Consumer i 's mean number of store brand purchases in category j , μ_{ij} , should vary directly with his or her total number of purchases in the product category c_{ij} .¹ Therefore, the consumer's category purchase rate appears as a scaling factor.

We assume that $b_i = (b_{i1}, b_{i2}, \dots, b_{iJ})'$ is distributed as a multivariate normal random variable and use a general variance-covariance structure to accommodate the correlation among the b_{ij} 's. That is,

$$b_i \sim N(0, \Sigma), \tag{3}$$

where Σ is an unrestricted $J \times J$ covariance matrix, with σ_{jk} denoting the (j, k) element of Σ and ρ_{jk} denoting the (j, k) element in the corresponding correlation matrix based on Σ . Thus, the covariance matrix Σ provides a measure of the interrelation between the propensities to purchase the store brand across different categories.

Let $y_i = (y_{i1}, y_{i2}, \dots, y_{iJ})'$ denote the store-brand purchase vector and $\mu_i = (\mu_{i1}, \mu_{i2}, \dots, \mu_{iJ})'$ denote the mean purchase rate vector. The model specification in Eqs. (1–3) implies that μ_i follows a multivariate lognormal distribution and y_i follows a multivariate Poisson-lognormal distribution. Then, the marginal means, variances, and

¹One alternative modeling approach would include c_{ij} as a covariate, because heavy buyers in a category may be more prone to buy store brands. In an estimation of this specification, the fit of the alternative model is significantly inferior to that of the proposed model. Therefore, c_{ij} serves as a scaling factor in this paper.

covariances of the store-brand purchase count distribution are as follows (Aitchison and Ho, 1989; Tunaru, 2002):

$$E[y_{ij}|\beta_j, \Sigma] = c_{ij} \exp(X'_{ij}\beta_j + 0.5\sigma_{jj}) = \alpha_{ij}, \quad j = 1, 2, \dots, J, \tag{4}$$

$$\text{Var}[y_{ij}|\beta_j, \Sigma] = \alpha_{ij} + \alpha_{ij}^2(\exp(\sigma_{jj}) - 1), \quad j = 1, 2, \dots, J, \tag{5}$$

and

$$\text{Cov}[y_{ij}, y_{ik}|\beta_j, \beta_k, \Sigma] = \alpha_{ij}\alpha_{ik}(\exp(\sigma_{jk}) - 1), \quad j \neq k. \tag{6}$$

The proposed model thus allows for overdispersion because the marginal variance of the count variable is greater than the marginal mean. The variance of the sample counts also is greater than the variance of the mean purchase rate. Hence, simply using sample correlation values would result in a downward bias in the estimates of the correlations of the mean parameters.²

The proposed multivariate Poisson-lognormal model provides a powerful and flexible tool to capture cross-category correlations in purchase count data. First, it can recover positive and negative correlations among the cross-category purchases of the store brand, depending on the sign of σ_{jk} in Eq. (6). Second, intuitively appealing special cases of the proposed model can be tested easily. For example, the hypotheses of independence among the store brand purchase counts across product categories can be tested by setting the appropriate off-diagonal elements σ_{jk} in the variance-covariance matrix Σ equal to 0 and comparing the fits of the proposed versus the constrained "independence" models. The independence model assumes the omitted factors are category-specific variables and do not have common effects on store brand purchasing across multiple product categories, which may occur when price sensitivities and/or value consciousness are category-specific characteristics and the quality perceptions of store brands are inconsistent across different categories. Third, when the correlation coefficients ρ_{jk} approach unity, b_{ij} and b_{ik} are common across categories, which indicates that the same unobserved consumer-specific factors underlie store brand purchasing behavior across categories. Therefore,

²The correlation of sample counts thus is smaller than the correlation of the mean parameters, due to the larger variances in measuring the count variables.

$$\begin{aligned} \text{Var}[y_{ij}] &= E[\text{Var}(y_{ij}|\mu_{ij})] + \text{Var}[E(y_{ij}|\mu_{ij})] = E[\mu_{ij}] + \text{Var}[\mu_{ij}], \quad j = 1, 2, \dots, J, \\ \text{Cov}[y_{ij}, y_{ik}] &= E[\text{Cov}(y_{ij}, y_{ik}|\mu_{ij}, \mu_{ik})] + \text{Cov}[E(y_{ij}|\mu_{ij}), E(y_{ik}|\mu_{ik})] \\ &= \text{Cov}[\mu_{ij}, \mu_{ik}], \quad j, k = 1, 2, \dots, J, \end{aligned}$$

$$\begin{aligned} |\text{Corr}(y_{ij}, y_{ik})| &= \left| \frac{\text{Cov}(y_{ij}, y_{ik})}{\sqrt{\text{Var}(y_{ij})\text{Var}(y_{ik})}} \right| = \left| \frac{\text{Cov}(\mu_{ij}, \mu_{ik})}{\sqrt{\text{Var}(y_{ij})\text{Var}(y_{ik})}} \right| \\ &< \left| \frac{\text{Cov}(\mu_{ij}, \mu_{ik})}{\sqrt{\text{Var}(\mu_{ij})\text{Var}(\mu_{ik})}} \right| = |\text{Corr}(\mu_{ij}, \mu_{ik})| \\ &= \left| \frac{c_{ij} \exp(X'_{ij}\beta_j + 0.5\sigma_{jj})(\exp(\sigma_{jk}) - 1)c_{ik} \exp(X'_{ik}\beta_k + 0.5\sigma_{kk})}{c_{ij} \exp(X'_{ij}\beta_j + 0.5\sigma_{jj})\sqrt{(\exp(\sigma_{jj}) - 1)(\exp(\sigma_{kk}) - 1)}c_{ik} \exp(X'_{ik}\beta_k + 0.5\sigma_{kk})} \right|. \end{aligned}$$

price sensitivity and/or value consciousness are household traits that have common effects on store brand purchasing behavior, and the perceived qualities of store brands relative to those of national brands remain consistent across categories. This constrained model thus represents the “one-factor” model.

3.2. Model estimation

The probability density function for the store-brand purchase count vector, which follows a multivariate Poisson-lognormal distribution, is given by

$$p(y_i|\boldsymbol{\beta}, \Sigma) = \int \prod_{j=1}^J f(y_{ij}|\beta_j, b_{ij})\phi_J(b_i|0, \Sigma) db_i. \quad (7)$$

The multiple integrals cannot be solved in a closed form for arbitrary Σ . Therefore, the MCMC method under the Bayesian framework can be used to compute the integrals. The constructed Markov chain ensures the stationary distribution of the chain is the joint posterior distribution, represented by the joint density, $\pi(\boldsymbol{\beta}, \Sigma|y)$. The random draws furnished by sampling the Markov chain after a burn-in stage serve as approximate draws from the posterior distribution. The long-run averages of the draws then can create simulation-based estimates. For the proposed multivariate Poisson model, the strategy for constructing the Markov chain is based on the work by Chib and Winkelmann (2001). The MCMC sampler uses a combination of Gibbs and Metropolis steps, and following the concept of data augmentation (Tanner and Wong, 1987), the parameter vector in the MCMC sampler includes the latent variable \mathbf{b} in addition to $\boldsymbol{\beta}$ and Σ . Given starting values, successive draws are made from the following sampling routine (for further discussion of the conditionals, see Chib and Winkelmann, 2001):

1. Draw $\mathbf{b}|y, \boldsymbol{\beta}, \Sigma$ using the Metropolis method.
2. Draw $\boldsymbol{\beta}|y, \mathbf{b}$ using the Metropolis method.
3. Draw $\Sigma^{-1}|\mathbf{b}$ from a Wishart distribution.
4. Repeat.

Inferences come from the last 1000 draws after discarding the draws of the initial 10,000 iterations.³

4. Data

We use data from a national warehouse club to calibrate the proposed model. The focal store brand carries a consistent name and logo in all categories, with the exception of the diaper category, where it is cobranded with a different label.⁴ It is a premium quality store brand

³Before applying the estimation to the store brand purchase data, a synthetic data set tests the robustness of the estimation procedure. The algorithm successfully recovers the true values of the parameters used to create the synthetic data.

⁴The packaging includes the same focal store brand and the same logo, as well as a different (more emphasized) brand name.

Table 1
Category and store brand purchase rates

| Category | Category/store brand purchase rate | | | |
|--------------|------------------------------------|-----------|------|-------|
| | Mean | Std. Dev. | Min. | Max. |
| Trash bags | 2.27/0.89 | 1.71/1.11 | 1/0 | 33/12 |
| Diaper | 5.72/1.82 | 6.44/4.04 | 1/0 | 43/38 |
| Detergent | 5.17/0.54 | 4.44/1.72 | 1/0 | 43/28 |
| Multivitamin | 1.77/0.69 | 1.21/1.00 | 1/0 | 11/8 |
| Ibuprofen | 1.82/0.70 | 1.41/0.98 | 1/0 | 14/8 |

according to industry reports (Frozen Food Age, 2000; Warehouse Club Focus, 1998). Because of this premium quality positioning, there is a higher likelihood of quality spillover effects.

The data span more than 60 weeks from August 1999 to October 2000 and include a nationally representative sample of households. The empirical investigation analyzes consumers' purchases of store brands in five categories: trash bags, diapers, liquid laundry detergent, multivitamins, and ibuprofen. To capture cross-category correlations, we select individual consumers who purchase at least once from each of the five categories, for a sample of 1743 households. The total numbers of purchases aggregated over households during the observation period for each category are as follows: 4015 (trash bags), 10,019 (diapers), 8993 (laundry detergent), 3085 (multivitamin), and 3205 (ibuprofen).

Table 1 displays the category and store-brand purchase rates in each of the five product categories. The mean category purchase rate is the total number of product purchases across households in each category divided by the total number of households in the sample. As Table 1 shows, the average store brand purchase rate is approximately one-third of the category purchase rate in four of the five product categories, whereas in the laundry detergent market, the store brand purchase rate is only one-tenth of the category purchase rate. In the multivitamin and ibuprofen categories, approximately 60% of the households purchased only once in the two categories. Because such limited time series information per household may lead to insignificant estimates of the correlations of brand preferences in a multiple-category brand choice model, purchases must be aggregated over time in multivariate data that involve infrequently purchased categories.

While 11% of the sample did not purchase the focal store brand in any of the five categories, the percentages of those that purchased a store brand from one, two, three, four, or all five categories are 25%, 28%, 20%, 13%, and 3%, respectively. Fig. 1 displays histograms of households with different numbers of store brand purchases for each of the five categories and displays the overdispersion through excess zeros (Cameron and Trivedi, 1998). The focal store brand has a market share of 42%, 32%, 10%, 38%, and 38% in the trash bag, diaper, liquid detergent, multivitamin, and ibuprofen categories, respectively, with the

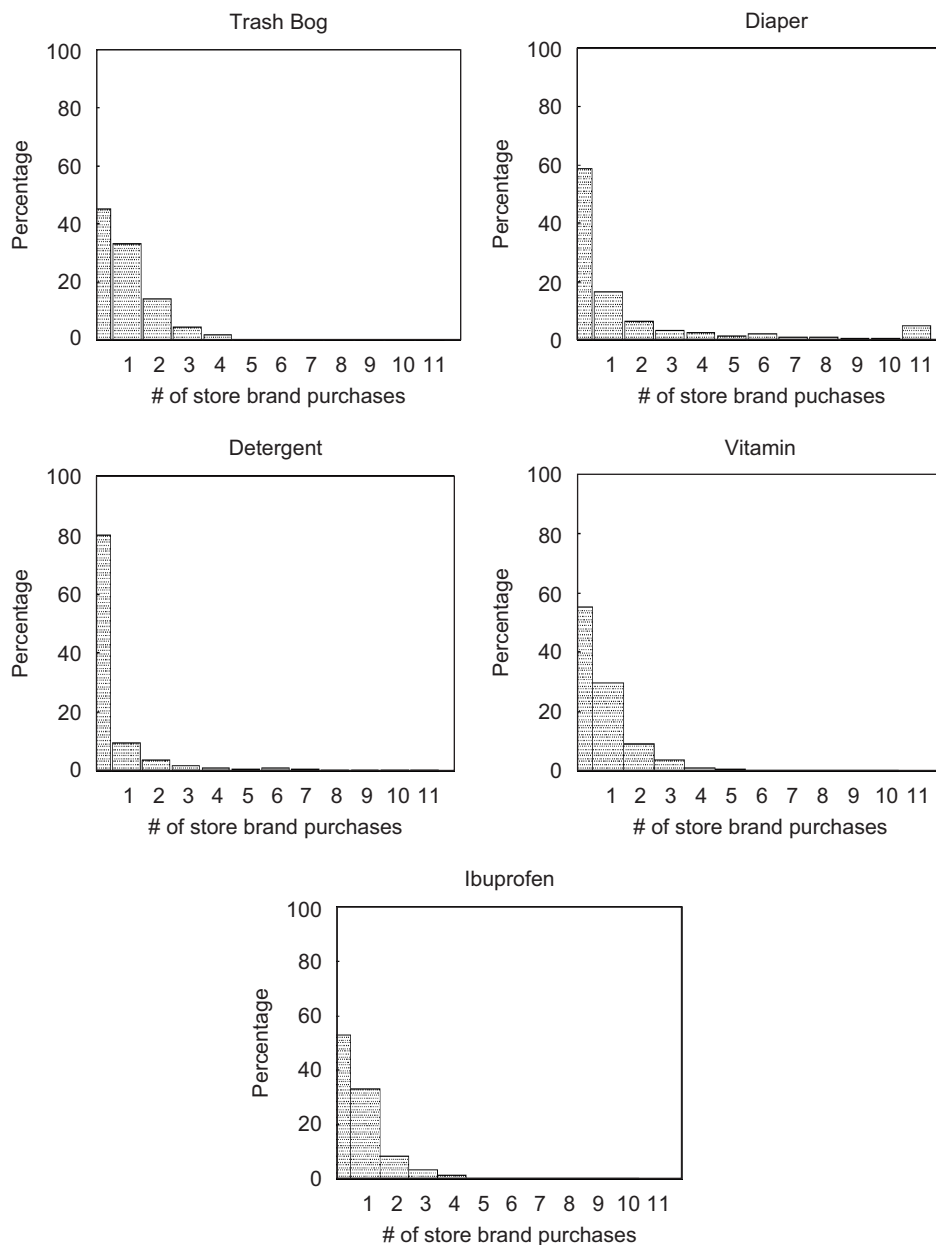


Fig. 1. Histograms of store brand purchases in five categories.

second highest market share in all the categories except laundry detergents. The laundry detergent market includes four national brands that share 90% of the market. In the multivitamin and ibuprofen categories, a national brand owns approximately two-thirds of the market. The trash bag category contains three national brands, and the diaper category has two national brands, with combined market shares of 58% and 68%, respectively.

Our proposed multivariate Poisson regression model includes four explanatory variables: (i) the price spread between the store brand and national brands, (ii) frequency of store visits, (iii) duration of a customer's membership in the warehouse club, and (iv) average basket size. Table 2 provides summary statistics for these four explanatory

variables. The operationalization of the price spread variable follows Hoch and Banerji's (1993) definition, that is, the logarithm of the percentage price difference of the store brand relative to the leading national brand. The length of a customer's relationship with the warehouse club outlet is measured by the time elapsed since the customer's club membership was initiated. The warehouse club rescales data pertaining to store visit frequency and total shopping expenditures. Hence, the measure of a customer's frequency of visits reflects the amount of times that the customer visited the store relative to other customers and the shopping basket measure reflects the customer's size of the shopping basket per store visit relative to other customers. Logarithms of the last three

Table 2
Descriptive statistics for the explanatory variables

| | Explanatory variables | | Descriptive statistics | |
|---|-----------------------|-----------|------------------------|---------|
| | Mean | Std. Dev. | Min. | Max. |
| Number of store visits | 45.35 | 31.72 | 2.00 | 221.00 |
| Basket size | 151.50 | 171.12 | 39.71 | 4348.30 |
| Duration of membership (yr) | 5.28 | 2.91 | 0.21 | 14.65 |
| Price spread: (national brand—store brand)/(national brand) | | | | |
| Trash bag | 0.31 | 0.10 | 0.17 | 0.52 |
| Diaper | 0.10 | 0.19 | −0.82 | 0.28 |
| Detergent | 0.16 | 0.03 | 0.04 | 0.29 |
| Multivitamin | 0.14 | 0.06 | 0.05 | 0.51 |
| Ibuprofen | 0.77 | 0.02 | 0.72 | 0.84 |

variables account for their diminishing effects on store-brand purchases.

A larger price spread should lead to more store-brand purchases, because a higher price discount reinforces the value positioning of a store brand. Consumers who make frequent trips to the store are likely to have greater loyalty toward it, and Ailawadi et al. (2001) find that store brand usage correlates positively with store loyalty. Therefore, we expect a positive relationship between store visit frequency and store brand purchasing propensity. It is likely that the longer-standing warehouse club members are more familiar with and have more confidence in the brand that the store carries, so we hypothesize a positive relationship between store brand purchasing propensity and membership duration. Finally, because customers with large versus small shopping baskets may be less responsive to price differences between brands in individual product categories (Bell and Lattin, 1998), the sign of the association between store brand purchase propensity and the size of the shopping basket is expected to be negative.

5. Results

5.1. Fit of the proposed versus alternative benchmark models

To assess the fit of the proposed multivariate Poisson-lognormal model, we compare the marginal distributions of the predicted counts with the observed count distributions. The marginal probabilities of the predicted and actual number of store brand purchases by category are displayed in Table 3. The predicted marginal probabilities are close to the corresponding observed values, indicating a good fit of the proposed multivariate Poisson-lognormal model.

Next, we compare the fit of the proposed model with that of selected benchmark models. The first benchmark model we consider is the univariate Poisson lognormal model (UVPLN) that assumes independence in consumers' propensities to purchase the store brand across different

product categories. In this constrained “independence model,” the off-diagonal elements in the variance–covariance matrix Σ equal 0. The second benchmark model is the one-factor multivariate Poisson lognormal model (MVPLN) with perfectly correlated unobserved heterogeneity. This model involves introducing a common log-normally distributed random variable into the Poisson mean function for each category. Our one-factor model with a log-normal mixing distribution is a variation of Munkin and Trivedi's (1999) multivariate negative binomial model (MVNB). In the MVNB model specification, which is also a one-factor model, the Poisson density of each category mixes with common unobserved gamma heterogeneity (Marshall and Olkin, 1990).⁵ A closed-form solution of the resulting one-factor MVNB model exists which makes the model amenable to maximum likelihood estimation. Both maximum likelihood and MCMC methods are used for the MVNB model.

We use deviance statistics to assess the goodness-of-fit of a model and compare the relative fit of the proposed model versus the alternative benchmark models (McCullagh and Nelder, 1989; Cameron and Trivedi, 1998).⁶ The deviance of the proposed model, 1140.3, is lower than the deviance values of the other benchmark models, which indicates that the proposed model provides a superior fit. Specifically, the independence model yields a significantly higher deviance of 1312.7. The one-factor benchmark model produces a marginally higher deviance of 1195.2, and the MVNB estimated by maximum likelihood and MCMC yields deviances of 10180.7 and 9974.8, respectively. The deviance statistics of the MVNB model are considerably higher than that of the one-factor benchmark model with a log-normal mixing distribution, which implies that the lognormal distribution provides a better representation of the unobserved heterogeneity than does the gamma distribution. As diagnostic evidence of the inferior fit of MVNB model on the deviance measure, Table 4 provides the results of a comparison of the marginal distributions of the predicted counts using MVNB model (MCMC) with observed count distributions. We observe that the MVNB model significantly underpredicts the percentage of “non-store brand buyers” in the diaper and detergent categories by approximately 20%, which clearly indicates its disadvantage in capturing the extra proportion of zero counts. Comparing maximum likelihood and MCMC methods with regard to MVNB model fit based on the deviance statistic, the MCMC estimates provide a slightly better fit, probably because MCMC can better handle the complex likelihood function of gamma mixture.

⁵The gamma variable is distributed with density: $f(v) = \frac{v^{z-1} \exp(-v)}{\Gamma(z)}$.

⁶The deviance of a fitted model compares the log-likelihood of the fitted model with the log-likelihood of a saturated model with n parameters that fits n observations perfectly. For the normal model, the deviance is the residual sum of squares; for the Poisson model, it is the statistic labeled G^2 by Bishop et al. (1975). The form of deviance used in this study is $2 \sum \{y \log(y/\hat{\mu}) - (y - \hat{\mu})\}$, where y is the data value, and $\hat{\mu}$ is the fitted value from the model (McCullagh and Nelder, 1989).

Table 3
Comparisons of observed and predicted marginal probabilities based on the proposed model

| Store brand purchases | Trash bag | | Diaper | | Detergent | | Multivitamin | | Ibuprofen | |
|-----------------------|-----------|-------|--------|-------|-----------|-------|--------------|-------|-----------|-------|
| | Act. | Pred. | Act. | Pred. | Act. | Pred. | Act. | Pred. | Act. | Pred. |
| $P(y = 0)$ | 0.45 | 0.47 | 0.59 | 0.55 | 0.80 | 0.79 | 0.55 | 0.56 | 0.53 | 0.56 |
| $P(y = 1)$ | 0.33 | 0.31 | 0.16 | 0.19 | 0.10 | 0.11 | 0.30 | 0.28 | 0.33 | 0.28 |
| $P(y = 2)$ | 0.14 | 0.13 | 0.06 | 0.08 | 0.04 | 0.04 | 0.09 | 0.10 | 0.08 | 0.10 |
| $P(y = 3)$ | 0.05 | 0.05 | 0.03 | 0.04 | 0.02 | 0.02 | 0.04 | 0.04 | 0.04 | 0.04 |
| $P(y > = 4)$ | 0.03 | 0.03 | 0.15 | 0.14 | 0.04 | 0.04 | 0.02 | 0.02 | 0.02 | 0.02 |

Table 4
Comparisons of observed and predicted marginal probabilities based on the multivariate negative binomial model (MCMC)

| Store brand purchases | Trash bag | | Diaper | | Detergent | | Multivitamin | | Ibuprofen | |
|-----------------------|-----------|-------|--------|-------|-----------|-------|--------------|-------|-----------|-------|
| | Act. | Pred. | Act. | Pred. | Act. | Pred. | Act. | Pred. | Act. | Pred. |
| $P(y = 0)$ | 0.45 | 0.46 | 0.59 | 0.40 | 0.80 | 0.61 | 0.55 | 0.56 | 0.53 | 0.53 |
| $P(y = 1)$ | 0.33 | 0.31 | 0.16 | 0.24 | 0.10 | 0.25 | 0.30 | 0.29 | 0.33 | 0.30 |
| $P(y = 2)$ | 0.14 | 0.14 | 0.06 | 0.13 | 0.04 | 0.09 | 0.09 | 0.10 | 0.08 | 0.11 |
| $P(y = 3)$ | 0.05 | 0.06 | 0.03 | 0.07 | 0.02 | 0.03 | 0.04 | 0.03 | 0.04 | 0.04 |
| $P(y > = 4)$ | 0.03 | 0.04 | 0.15 | 0.16 | 0.04 | 0.02 | 0.02 | 0.02 | 0.02 | 0.02 |

5.2. Predictive ability of the proposed versus alternative models

To further assess the fit of the proposed versus the alternative benchmark models, we examine their relative ability to predict cross-category store brand purchasing behavior by constructing different patterns on the basis of a dichotomy of store-brand purchasing and identifying those households that follow a certain pattern. For each category, a value of 0 indicates no store brand purchase observations, whereas a value of 1 indicates one or more store brand purchases. For example, the purchase pattern (1, 0, 0, 0, 0) means that the household has purchased the store brand in the first category but not in the other four categories. The five different categories entail 32 different purchase patterns, which can be numbered such that l represents the purchase pattern and equals 1–32. Each of the 1743 individual households follows a specific purchase pattern, so the $N_{l,\text{predicted}}$ and $N_{l,\text{actual}}$ values represent the numbers of households that follow pattern l , identified using a specified model (i.e., the subscript predicted) or an actual observation. The hit rate formula $1 - (\sum_{l=1}^{32} |N_{l,\text{predicted}} - N_{l,\text{actual}}| / n)$ lies between 0 and 1 (perfect predictability) and provides an assessment of the performance of each model. The proposed model has the highest hit rate, 79%, indicating that it predicts the joint probabilities better than the alternate independence, one-factor, and the two MVNB models (see Table 5). For example, the independence model (UVPLN), which does not account for cross-category correlations, yields a lower hit rate of 70%. Even out-of-sample predictions provide stronger

support for the importance of cross-category correlations in propensity to purchase the store brand.⁷ Therefore, these results indicate that there is a significant correlation exists in consumer propensity to purchase the focal store brand across different product categories.

The hit rate of the one-factor model (MVPLN) is substantially below that of the proposed model, at 0.59 compared with 0.79 (see Table 5). The one-factor MVPLN model performs even worse than the independence model (UVPLN)—0.59 versus 0.70. A key reason for this inferior performance on the hit rate measure likely pertains to the one-factor model’s substantial misprediction of the store brand purchases that are devoted *solely* to one category (see Table 4). That is, the homogeneous correlation structure imposed by the one-factor model cannot properly accommodate the group of sole store brand purchasers in each category, who collectively represent about 25% of the total buyers. Finally, the MVNB model, which is also a one-factor model, has the worst hit rates at 0.47 (ML) and 0.48 (MCMC), possibly because of its inadequacy in terms of capturing excess zero counts in the data, as discussed previously. The prediction of joint probabilities involving zero counts in the purchase pattern exacerbates the issue.

⁷To calculate the out-of-sample performance of the proposed model, the evenly split data provide estimations of the respective model parameters using the first 900 observations. The other half of the data offers the model predictions. The hit rates for the proposed model, which accounts for the correlations, and the independence model are 77% and 66%, respectively.

Table 5
Event prediction of proposed versus alternative models

| Type | Event ^a | Observed no. of HH's | Proposed model | Independence model | One-factor model | MVNB model (ML) | MVNB model (MCMC) |
|----------|--------------------|----------------------|----------------|--------------------|------------------|-----------------|-------------------|
| 1 | (0, 0, 0, 0, 0) | 195 | 174 | 128 | 187 | 73 | 86 |
| 2 | (1, 0, 0, 0, 0) | 175 | 147 | 129 | 91 | 72 | 79 |
| 3 | (0, 1, 0, 0, 0) | 74 | 93 | 95 | 126 | 97 | 105 |
| 4 | (0, 0, 1, 0, 0) | 20 | 24 | 27 | 46 | 33 | 37 |
| 5 | (0, 0, 0, 1, 0) | 71 | 83 | 87 | 63 | 48 | 52 |
| 6 | (0, 0, 0, 0, 1) | 90 | 94 | 92 | 74 | 58 | 63 |
| 7 | (1, 1, 0, 0, 0) | 99 | 102 | 102 | 88 | 106 | 108 |
| 8 | (1, 0, 1, 0, 0) | 26 | 26 | 30 | 32 | 37 | 38 |
| 9 | (1, 0, 0, 1, 0) | 73 | 79 | 94 | 44 | 51 | 52 |
| 10 | (1, 0, 0, 0, 1) | 88 | 89 | 98 | 52 | 61 | 62 |
| 11 | (0, 1, 1, 0, 0) | 23 | 19 | 24 | 41 | 49 | 50 |
| 12 | (0, 1, 0, 1, 0) | 38 | 60 | 70 | 58 | 70 | 71 |
| 13 | (0, 1, 0, 0, 1) | 41 | 58 | 67 | 61 | 74 | 75 |
| 14 | (0, 0, 1, 1, 0) | 11 | 16 | 21 | 23 | 26 | 26 |
| 15 | (0, 0, 1, 0, 1) | 10 | 20 | 21 | 25 | 28 | 29 |
| 16 | (0, 0, 0, 1, 1) | 85 | 61 | 69 | 37 | 42 | 43 |
| 17 | (1, 1, 1, 0, 0) | 16 | 26 | 27 | 43 | 60 | 58 |
| 18 | (1, 1, 0, 1, 0) | 57 | 75 | 80 | 65 | 82 | 78 |
| 19 | (1, 1, 0, 0, 1) | 52 | 70 | 74 | 66 | 85 | 81 |
| 20 | (1, 0, 1, 1, 0) | 15 | 21 | 25 | 23 | 30 | 29 |
| 21 | (1, 0, 1, 0, 1) | 16 | 24 | 25 | 25 | 33 | 32 |
| 22 | (1, 0, 0, 1, 1) | 84 | 66 | 78 | 38 | 48 | 45 |
| 23 | (0, 1, 1, 1, 0) | 10 | 17 | 20 | 29 | 40 | 39 |
| 24 | (0, 1, 1, 0, 1) | 9 | 18 | 18 | 29 | 40 | 39 |
| 25 | (0, 1, 0, 1, 1) | 54 | 52 | 53 | 45 | 58 | 55 |
| 26 | (0, 0, 1, 1, 1) | 31 | 18 | 18 | 18 | 24 | 23 |
| 27 | (1, 1, 1, 1, 0) | 20 | 27 | 24 | 50 | 54 | 49 |
| 28 | (1, 1, 1, 0, 1) | 27 | 27 | 21 | 46 | 53 | 48 |
| 29 | (1, 1, 0, 1, 1) | 119 | 72 | 64 | 74 | 72 | 65 |
| 30 | (1, 0, 1, 1, 1) | 33 | 27 | 23 | 28 | 31 | 28 |
| 31 | (0, 1, 1, 1, 1) | 22 | 22 | 16 | 33 | 37 | 34 |
| 32 | (1, 1, 1, 1, 1) | 59 | 39 | 22 | 82 | 54 | 47 |
| Total | | 1743 | 1743 | 1743 | 1743 | 1743 | 1743 |
| Hit rate | | | 0.79 | 0.70 | 0.59 | 0.47 | 0.48 |

^aA value of 1 indicates store brand purchase, and 0 indicates no store brand purchase. The respective order of the categories in the parentheses is trash bag, diaper, laundry detergent, multivitamin, and ibuprofen.

5.3. Estimates of correlations of store brand purchase propensities across product categories

Table 6 displays the posterior estimates of correlations in consumers' propensities to purchase the store brand across different categories on the basis of the proposed multivariate Poisson-lognormal model. The estimated correlations of latent effects are all positive and substantial. The 90% posterior interval *excludes* 0 in all estimates (Gelman et al., 1995). As expected, the magnitude of the correlation between the similar categories of multivitamins and ibuprofen is high (0.97). However, even dissimilar product category pairs, such as diapers and trash bags, exhibit high correlations in store brand purchasing propensities that exceed 0.80. The correlations of the purchasing propensity in the laundry detergent category with those of other categories, though high, are relatively lower than the correlations among other category pairs.

Table 6
Estimates of correlations in store brand purchase propensities

| | Trash bag | Diaper | Detergent | Multivitamin | Ibuprofen |
|--------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------|
| Trash bag | 1.00 | | | | |
| Diaper | 0.81 ^a (0.05) | 1.00 | | | |
| Detergent | 0.62 ^a (0.09) | 0.30 ^a (0.06) | 1.00 | | |
| Multivitamin | 0.83 ^a (0.08) | 0.50 ^a (0.07) | 0.44 ^a (0.09) | 1.00 | |
| Ibuprofen | 0.83 ^a (0.06) | 0.43 ^a (0.08) | 0.57 ^a (0.09) | 0.97 ^a (0.02) | 1.00 |

^aEstimates have more than 90% of the posterior mass away from 0. Figures in parentheses are posterior standard deviations of correlation coefficients.

The observed estimates of high positive correlations across related and even unrelated categories may be due to the premium quality positioning of the focal store brand

and the spillover effects of the umbrella branding strategy adopted by the warehouse club. These activities improve quality perceptions of the store brand and enhance its position among intermediate segment consumers, that is, those who are willing to trade off between price and quality. Hence, a store brand’s appeal across categories may arise from its acceptable quality rather than its lower price. The high correlation estimates in store brand purchase propensities even across unrelated categories suggests that attribute similarity has limited impact in deterring the quality transfer process in the context of store brands.

It is possible that the high correlation estimates are due to shared low store brand purchase propensities among households who exclusively purchase national brands (indicated by excess “zeros” in data). To further investigate the nature of the correlations, we compute cross-category conditional purchasing probabilities of store brand versus national brand using the model estimates on the joint distribution. The conditional probabilities are compared against the marginal probabilities of the store versus national brand purchasing in each category. If the conditional probability is higher than the marginal probability, it would imply that purchasing store (national) brand in one category makes the purchase of store (national) brand in another more likely. As before, let y_{ij} represent the number of store brand purchases for household i in category j . The conditional probability of purchasing a store brand in category k given purchasing

a store brand in category j can be expressed as $P(y_{ik} > 0 | y_{ij} > 0)$ and the marginal probability for comparison is $P(y_{ik} > 0)$. Likewise, the conditional probability of purchasing national brand exclusively in category k given exclusive national brand purchases in category j is $P(y_{ik} = 0 | y_{ij} = 0)$ and the marginal probability is $P(y_{ik} = 0)$. Note that the conditional probability pattern of store brand (shown in Table 7) does not necessarily parallel the correlation pattern (shown in Table 6), because the strength of the correlation also reflects differences in purchase amount of the store brand across categories, whereas the conditional probability simply includes the overall incidence of store brand purchases during the observation period.

Tables 7 and 8 present the respective cross-category conditional likelihoods of store brand and national brand purchasing for all category pairs as well as the marginal probabilities that serve as comparison points. The results show that most conditional purchasing probabilities appear to be higher than the marginal probabilities with respect to both the store brand and the national brand. Hence, the estimated high correlations in store brand purchase propensities across categories (see Table 6) may be attributed in part to the shared low store brand purchasing propensities among some households who buy exclusively national brands across categories. Even so, the cross-category conditional likelihoods of store brand purchasing in Table 7 provide evidence of positive association in store brand purchasing tendencies across

Table 7
Cross-category store brand (SB) purchasing likelihood in category k , given purchase of SB in category j

| Conditional probability of SB purchasing in category k $P(y_k > 0 y_j > 0)$ given SB purchasing in category j | Trash bag | Diaper | Detergent | Multivitamin | Ibuprofen |
|---|-----------|--------|-----------|--------------|-----------|
| <i>Trash bag</i> | NA | 0.47 | 0.23 | 0.44 | 0.45 |
| <i>Diaper</i> | 0.55 | NA | 0.25 | 0.47 | 0.46 |
| <i>Detergent</i> | 0.56 | 0.53 | NA | 0.50 | 0.47 |
| <i>Multivitamin</i> | 0.55 | 0.50 | 0.25 | NA | 0.48 |
| <i>Ibuprofen</i> | 0.55 | 0.47 | 0.26 | 0.47 | NA |
| Marginal probability $P(y_k > 0)$ | 0.53 | 0.45 | 0.21 | 0.44 | 0.44 |

Note that the 90% posterior intervals of all conditional probabilities exclude the values of marginal probabilities in the last row that serve as comparison points except that of $P(\text{multivitamin} | \text{trash bag})$.

Table 8
Cross-category national brand (NB) purchasing likelihood in category k , given purchase of NB in category j

| Conditional probability of NB purchasing in Category k $P(y_k = 0 y_j = 0)$ given NB purchasing in category j | Trash bag | Diaper | Detergent | Multivitamin | Ibuprofen |
|---|-----------|--------|-----------|--------------|-----------|
| <i>Trash bag</i> | NA | 0.59 | 0.81 | 0.60 | 0.58 |
| <i>Diaper</i> | 0.50 | NA | 0.82 | 0.62 | 0.59 |
| <i>Detergent</i> | 0.48 | 0.58 | NA | 0.60 | 0.60 |
| <i>Multivitamin</i> | 0.49 | 0.59 | 0.82 | NA | 0.61 |
| <i>Ibuprofen</i> | 0.48 | 0.57 | 0.83 | 0.62 | NA |
| Marginal probability $P(y_k = 0)$ | 0.47 | 0.55 | 0.79 | 0.56 | 0.56 |

Note that the 90% posterior intervals of all conditional probabilities exclude the values of marginal probabilities in the last row that serve as comparison points.

Table 9
Estimates of explanatory variable coefficients

| | Trash bag | Diaper | Detergent | Multivitamin | Ibuprofen |
|-----------------------|--------------------------------|--------------------------------|--------------------------------|-------------------------------|-------------------------------|
| Constant | −0.574 ^a (0.096) | −1.043 ^a (0.126) | −2.678 ^a (0.633) | −0.090 (0.183) | −0.164 (0.364) |
| Price discount | 0.319 ^a (0.019) | 0.482 ^a (0.062) | 0.582 ^a (0.344) | 0.521 ^a (0.096) | 3.424 ^a (1.372) |
| Store visit frequency | 0.053 (0.042) | 0.279 ^a (0.074) | 0.371 ^a (0.127) | 0.246 ^a (0.053) | 0.093 ^a (0.052) |
| Length of membership | −0.054 (0.036) | −0.094 (0.061) | 0.031 (0.102) | −0.000 (0.047) | −0.032 (0.045) |
| Basket size | −0.182 ^a (0.064) | 0.137 (0.121) | −0.114 (0.183) | 0.074 (0.083) | −0.027 (0.080) |

^aEstimates have more than 90% of the posterior mass away from 0. Figures in parentheses are posterior standard deviations of coefficient estimates.

categories because the posterior 90% intervals of the conditional likelihoods of store brand purchasing nearly exclude all the benchmarking marginal probabilities. This indicates that consumers' propensity to purchase the focal store brand is not driven by category-specific factors alone but also by consumer-specific characteristics. Some consumers who are price sensitive may seek out a store brand across categories; other consumers who are willing to trade off quality for a price discount may be willing to purchase a premium quality store brand in different product categories.

A comment or two also is in order regarding the observed lower cross-correlations for the laundry detergent category and other categories. This result may be because the perceived quality of the store brand is relatively low in the highly differentiated laundry detergent category. Recall that four national brands in this category constitute a 90% share of detergent purchases among shoppers at the warehouse club, and store brand laundry detergent purchases represent just one-tenth of the category purchase rate.

5.4. Estimates of explanatory variable coefficients

Table 9 displays explanatory variable coefficient estimates in each of the five categories. First, as expected, the sign of the store versus national brand price spread variable coefficient is positive in all five categories, which indicates that the propensity to purchase the store brand increases with the price discount offered by the store brand relative to the leading national brand. Second, also as expected, store visit frequency is positively associated with store brand purchase propensity in four of the five categories, which may reflect greater awareness of the store brand among more frequent shoppers at the warehouse club (Ailawadi et al., 2001). Third, the length of club membership does not have a significant impact on store brand purchase propensity in any of the five categories (the 90% posterior interval includes 0). Thus, it appears newer versus longer-standing club members do not perceive a higher risk in purchasing store brands, possibly because of the relative newness of the store-brand program, which launched

approximately one and half year before the sample period. Fourth, with regard to the size of the shopping basket, the posterior mean is negative and has more than 90% mass away from zero in the trash bag category. However, it is not consistently negative in other categories, whose 90% posterior interval estimates include zero. Therefore, the relationship between store brand purchases and basket size may be category specific.

6. Managerial implications

Store brand managers who aim to maximize store profits can plan joint promotions across different categories. For example, high volume, low margin items might be co-promoted with high margin, low volume items with strong associations. Using the proposed model, managers can assess and quantify the overall profit impact of jointly promoting a store brand with different combinations of categories. Specifically, two components are required to analyze the profitability of a joint promotion: (i) conditional purchasing likelihood and (ii) expected purchase quantity.

The computation of the first component, namely, the conditional purchasing likelihood is discussed in Section 5.3. With regard to the second component, the expected purchase quantity may be obtained by computing the conditional expectation of store brand purchases in one category given the number of purchases in another. Suppose n_j and n_k represent the number of purchases in categories j and k , respectively.⁸ Then, the conditional expectation is expressed as

$$E(y_{ik}|y_{ij} > 0) = \sum_{y=1}^{n_j} E(y_{ik}|y_{ij} = y),$$

where

$$E(y_{ik}|y_{ij} = y) = \sum_{c=1}^{n_k} c \cdot P(y_{ik} = c|y_{ij} = y). \tag{8}$$

⁸For illustration purpose, 4 serves as the maximum value for n_j and n_k in this analysis.

Table 10 presents a list of expectations of conditional purchasing quantity, demonstrating that the conditional quantity generally increases as the number of purchases in the focal category increases.

If a focal category is strategically selected for a store brand promotion, one relevant question is to determine the potential profit impact of jointly promoting the store brand in other categories. A profitability analysis of a joint

Table 10
Expected number of conditional store brand purchases

| Focal category <i>j</i> | | Expected number of conditional store brand purchases $E(y_k y_j)$ | | | | |
|-------------------------|--------------|---|--------|-----------|--------------|-----------|
| | | Trash bag | Diaper | Detergent | Multivitamin | Ibuprofen |
| <i>Trash bag</i> | <i>Y</i> = 1 | NA | 1.0455 | 0.4069 | 0.6725 | 0.6878 |
| | <i>Y</i> = 2 | NA | 1.1753 | 0.4784 | 0.7348 | 0.7284 |
| | <i>Y</i> = 3 | NA | 1.2938 | 0.5610 | 0.8010 | 0.7680 |
| | <i>Y</i> = 4 | NA | 1.4288 | 0.7076 | 0.9115 | 0.8353 |
| <i>Diaper</i> | <i>Y</i> = 1 | 0.8827 | NA | 0.4346 | 0.6820 | 0.7050 |
| | <i>Y</i> = 2 | 0.9346 | NA | 0.5051 | 0.7671 | 0.7288 |
| | <i>Y</i> = 3 | 0.9626 | NA | 0.5452 | 0.8290 | 0.7393 |
| | <i>Y</i> = 4 | 1.0752 | NA | 0.5706 | 0.9807 | 0.7211 |
| <i>Detergent</i> | <i>Y</i> = 1 | 0.9616 | 1.1871 | NA | 0.8011 | 0.8110 |
| | <i>Y</i> = 2 | 1.0457 | 1.3001 | NA | 0.9174 | 0.9372 |
| | <i>Y</i> = 3 | 1.0980 | 1.3839 | NA | 0.9855 | 1.0198 |
| | <i>Y</i> = 4 | 1.1615 | 1.4695 | NA | 1.0259 | 1.1174 |
| <i>Multivitamin</i> | <i>Y</i> = 1 | 0.8836 | 1.0908 | 0.4389 | NA | 0.7212 |
| | <i>Y</i> = 2 | 0.9535 | 1.2677 | 0.5602 | NA | 0.8606 |
| | <i>Y</i> = 3 | 1.0302 | 1.4214 | 0.6797 | NA | 0.9907 |
| | <i>Y</i> = 4 | 1.1408 | 1.6229 | 0.8369 | NA | 1.1755 |
| <i>Ibuprofen</i> | <i>Y</i> = 1 | 0.8845 | 1.0785 | 0.4405 | 0.7104 | NA |
| | <i>Y</i> = 2 | 0.9420 | 1.1629 | 0.5640 | 0.8439 | NA |
| | <i>Y</i> = 3 | 0.9936 | 1.2156 | 0.6792 | 0.9762 | NA |
| | <i>Y</i> = 4 | 1.0668 | 1.2785 | 0.8698 | 1.1994 | NA |

Table 11
Revenue implications for joint cross-category promotions

| Focal category <i>j</i> under promotion | Conditional revenues of joint promotion with category <i>k</i> | | | | |
|--|--|--------|-------------------|--------------|-----------|
| | Trash Bag | Diaper | Laundry Detergent | Multivitamin | Ibuprofen |
| <i>Trash bag</i> | | | | | |
| Purchasing likelihood | NA | 0.47 | 0.23 | 0.44 | 0.45 |
| Expected number of store brand purchases | NA | 1.12 | 0.46 | 0.71 | 0.71 |
| \$ sales per unit | NA | 24.99 | 12.78 | 10.02 | 9.94 |
| Revenue per buyer | NA | 13.07 | 1.32 | 3.15 | 3.17 |
| <i>Diaper</i> | | | | | |
| Purchasing likelihood | 0.55 | NA | 0.25 | 0.47 | 0.46 |
| Expected number of store brand purchases | 0.97 | NA | 0.50 | 0.82 | 0.72 |
| \$ sales per unit | 8.88 | NA | 12.78 | 10.02 | 9.94 |
| Revenue per buyer | 4.77 | NA | 1.62 | 3.88 | 3.29 |
| <i>Detergent</i> | | | | | |
| Purchasing likelihood | 0.56 | 0.53 | NA | 0.50 | 0.47 |
| Expected number of store brand purchases | 1.03 | 1.29 | NA | 0.89 | 0.92 |
| \$ sales per unit | 8.88 | 24.99 | NA | 10.02 | 9.94 |
| Revenue per buyer | 5.16 | 16.87 | NA | 4.42 | 4.32 |
| <i>Multivitamin</i> | | | | | |
| Purchasing likelihood | 0.55 | 0.50 | 0.25 | NA | 0.48 |
| Expected number of store brand purchases | 0.92 | 1.18 | 0.50 | NA | 0.79 |
| \$ sales per unit | 8.88 | 24.99 | 12.78 | NA | 9.94 |
| Revenue per buyer | 4.50 | 14.65 | 1.63 | NA | 3.82 |
| <i>Ibuprofen</i> | | | | | |
| Purchasing likelihood | 0.55 | 0.47 | 0.26 | 0.48 | NA |
| Expected number of store brand purchases | 0.91 | 1.11 | 0.50 | 0.78 | NA |
| \$ sales per unit | 8.88 | 24.99 | 12.78 | 10.02 | NA |
| Revenue per buyer | 4.41 | 13.16 | 1.66 | 3.69 | NA |

promotion between a focal category and others requires leveraging the two components discussed previously.⁹ The specific components used and the profit analysis results appear in Table 11. For illustration purposes, diapers serve as the focal category for the store-brand promotion, because of its high purchase frequency and the strength of the store brand. The profit analysis reveals that the profit of co-promoting the store brand in the trash bag category is approximately three times that of co-promoting the store-brand detergent. This can be partly attributed to differences in the conditional purchase incidence and conditional purchase quantity. By considering both elements, managers can better understand the source and nature of the association of store brand purchasing behavior across categories.

7. Conclusions

Data mining methods are often used to analyze purchase interdependence based on cross-category transaction data. This paper proposes a different way to analyze such data by focusing on interdependence of brand purchasing propensities across categories. Specifically, we develop a multivariate mixed Poisson-lognormal model in a Bayesian framework to investigate cross-category store brand purchasing behavior of consumers belonging to a warehouse club. The model provides flexibility in capturing cross-category correlations for sparse purchases associated with infrequently purchased categories or infrequent visits to retail channels such as warehouse clubs. The proposed model outperforms the benchmark models in terms of goodness of fit and prediction. We observe high correlations in consumers' latent propensities to purchase the store brand across not only two health care categories but also across dissimilar categories such as diapers, trash bags, and laundry detergent. Furthermore, we find that the price spread between the store brand and national brands and the frequency of store visits are positively associated with consumers' propensity to purchase the store brand.

These results have several implications for retail store managers. The findings imply that some consumers may transfer their quality perceptions of a premium quality store brand to related and even unrelated categories. However, these findings are based on the purchase behavior of members of a warehouse club; managers in discount chains such as Target may find that the equity of a store brand transfers to related categories but not to unrelated categories. In any case, knowing the magnitude and source of the correlations in cross-category store brand purchasing behavior helps managers plan store brand promotions across categories and store merchandising activities. For example, in the empirical application, this paper shows that the store brand does not have a strong position in the laundry detergent category, but the focal warehouse club managers may be able to stimulate its sales

through joint promotions and joint displays of the store brand in the laundry detergent and one or more other popular categories.

In terms of directions for future research, this study can be extended in several ways. Consumers who join warehouse clubs tend to be value conscious and thus likely more prone to buy the store brand to take advantage of price discounts. Therefore, it may be worthwhile to calibrate the proposed multivariate Poisson-lognormal model across different retail formats, such as discount stores and supercenters. An extended cross-category analysis also could consider the long-term influence of store-brand promotions by examining the effects of promoting the store brand in one category on the sales of the store brand and national brands in other categories. Furthermore, research should investigate whether and how promotion elasticities depend on the categories and retailers if cross-retailer data are available.

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⁹Profit margins are assumed to be similar across categories.

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