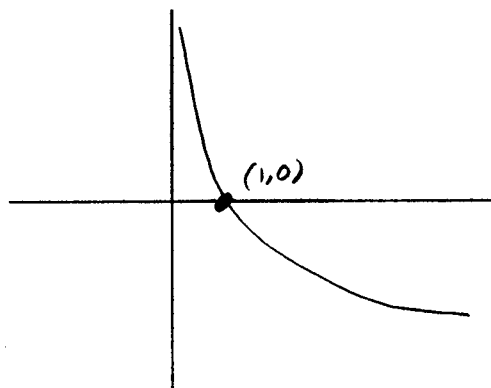
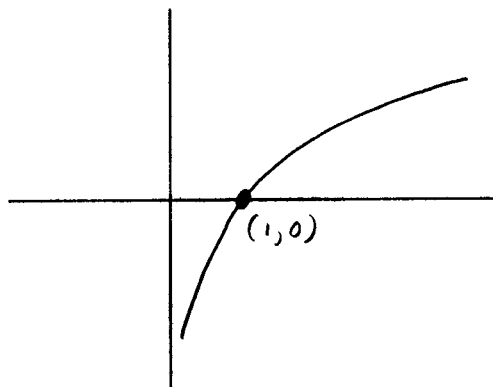


P-40 Sketch each logarithmic function.

1. $y = \log_a(x)$ where $a > 1$

2. $y = \log_a(x)$ where $0 < a < 1$



For each logarithm, we have been careful to indicate the *base* just after and slightly below the word *log*. Some bases are so often used that special notations are used for them. The word *log* without any base means *common logarithm* for which the base is 10. So $\log(100)$ means $\log_{10}(100)$ and thus $\log(100) = 2$. Find each of the following common logarithms.

3. $\log(10) = 1$ (since $10^1 = 10$)

5. $\log\left(\frac{1}{100}\right) = -2$ (since $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$)

4. $\log(1000) = 3$ (since $10^3 = 1000$)

6. $\log(1) = 0$ (since $10^0 = 1$)

The most important base for logarithms is the special number e (which we met on P-36) and \ln or *natural logarithm* means the logarithm to base e . Thus $\ln(e^2)$ means $\log_e(e^2)$ and so $\ln(e^2) = 2$. Find each of the following natural logarithms.

7. $\ln(e) = 1$ (since $e^1 = e$)

9. $\ln\left(\frac{1}{e^2}\right) = -2$ (since $e^{-2} = \frac{1}{e^2}$)

8. $\ln(\sqrt{e}) = \frac{1}{2}$ (since $e^{\frac{1}{2}} = \sqrt{e}$)

10. $\ln(1) = 0$ (since $e^0 = 1$)

Musicians use the *decibel* scale to express the relative loudness between two sounds by taking ten times the common logarithm of the power ratio between them. If an electric guitar puts out 1000 times more power than an acoustic guitar then its loudness is 30 dB greater since the common logarithm of 1000 is 3 and $10 \cdot 3 = 30$.

11. The Old-Fogy-Hearing-Helper amplifies sounds 10,000 times. How many dB does this represent?

$$\log(10\,000) = 4 \quad \text{so} \quad 10 \cdot 4 = \underline{\underline{40 \text{ dB}}}$$

12. A large truck 50 feet away makes a noise of about 90 dB while a jet plane landing 400 feet away has a loudness of 100 dB. How much more powerful is the noise of the plane?

10 dB more means the common logarithm of power ratio is 1 more so 10^1 times as powerful
(no: each +10 dB corresponds to being 10 times more powerful!)